International Journal of Theoretical Physics, Vol. 46, No. 7, July 2007 (© 2007)
DOI: 10.1007/s10773-006-9308-8

Quantum Dynamics of a Raman Atom Laser by Using a Feshbach-Resonance-Tuned Atomic Bose-Einstein Condensate

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Received September 24, 2006; accepted November 16, 2006
Published Online: February 8, 2007

We study a Raman atom laser output coupler by considering a Feshbach-resonance-tuned atomic Bose-Einstein condensate which is often viewed as a dilute system since the collisions can be tuned. However, the dimer formation can be triggered by using a Feshbach technique in a magnetic trap. We examine the quantum dynamics and statistics of this system analytically, including the quadrature-squeezed effects and the mutual coherence of the output atoms and the formed dimers.

KEY WORDS: atom laser; atom-molecule coherence; Bose-Einstein condensate.

The realization of Bose-Einstein condensates (BEC) in ultracold atomic gases has led to a profound revolution in modern physics, from low-temperature physics to atom optics (Anglin and Ketterle, 2002; Dalfovo *et al.*, 1999; Meystre, 2001). One important issue of recent efforts is of creating an atom laser and exploring its novel properties. Since the MIT-group first realized a pulsed atom laser by using quantum state transfer technique (Mewes *et al.*, 1997), many experimental achievements have been obtained in the design and amplification of an atom laser, which have also attracted great theoretical interests in probing both the outcoupling scheme and the rich properties of an atom laser system (Hagley *et al.*, 2001; Moy *et al.*, 1997; Ruostekoski *et al.*, 2003; Zhang *et al.*, 2003a). As an analogy of an optical laser, the atom laser is now expected to also have important applications in practice, such as a precise matter-wave interferometry (Abo-Shaeer *et al.*, 2005; Chapman *et al.*, 1995).

In the original work of Mewes *et al.* on a pulse atom laser (Mewes *et al.*, 1997), an attenuated RF field was applied to a dilute atomic BEC and the coherence of the output beam is ensured by a factorized structure of wave function of this

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light-atom system (Jing *et al.*, 2001). The nonlinear atomic collisions, however, can induce some nonclassical effects, such as a squeezed BEC experimentally realized by Orzel *et al.* (2001). Theoretical analysis fully including the role of collisions may need some extremely complicated methods, hence they are often neglected in probing an atom laser system by considering a sufficiently dilute condensate or tuning the atomic interactions with a Feshbach resonance (FR) method (Haine *et al.*, 2006). A dilute BEC sample is insufficient to obtain a bright or high-flux atom laser. While a FR-tuned BEC can be viewed as an ideal atom-laser source, the FR sweep itself can inevitably induce the coherent dimer creation (Holland *et al.*, 2001; Kokkelmans *et al.*, 2001; Timmermans *et al.*, 1999; van Abeelen and Verhaar, 1999).

The exitance of a hybrid atom-dimer BEC created via an atomic FR technique has been reported recently in a Rubidium or Cesium atomic condensate (Donley *et al.*, 2002; Herbig *et al.*, 2003; Inouye *et al.*, 1998; Roberts *et al.*, 2001). The quantum coherence of the atom-dimer coupling and the ability to detect dimers within a hybrid sample have been also demonstrated. In order to spatially separate the resulting dimers from the atoms, Herbig *et al.* designed a levitation trap with which they firstly obtained a pure diatomic molecular BEC by applying a Feshbach sweep and then a spatial Stern-Gerlach separation of the two species (Donley *et al.*, 2002; Herbig *et al.*, 2003; Inouye *et al.*, 1998; Roberts *et al.*, 2001). In the paper, we study a Raman atom laser output coupler by using a FR-tuned atomic BEC and focus on the role of FR-sweep-induced dimer formation on the quantum dynamics of the Raman atomic coupler. We show that this system can be adiabatically reduced into an effective three-state model for which the effects of quantum quadrature-squeezing and atom-dimer mutual coherence are observed. Also, as an interesting parallel of the scheme of Herbig *et al.*, (2003; Donlet *et al.*, 2002; Inouye *et al.*, 1998; Roberts *et al.*, 2001) this may shed a new light on another feasible method to separate the hybrid atom-dimer sample (Mackie *et al.*, 2000; Vanhaecke *et al.*, 2002; Winkler *et al.*, 2005; Wynar *et al.*, 2000) by utilizing a Raman-type output coupler. Finally we propose that, by applying a quantized optical photo-association (PA) field instead of a magnetic Feshbach sweep, a novel quantum transfer process and hence an atom-dimer entanglement may be expected.

As illustrated in Fig. 1, a large number of Bose-condensed atoms with three states are prepared initially in a trapped state $|1\rangle$. State $|2\rangle$, which is typically unconfined, is coupled to state $|1\rangle$ by an optical Raman transition with an intermediate state $|e\rangle$. The Feshbach sweep in the trap region can induce a resonantly enhanced atom-dimer coupling which coherently converts the atoms into weakly bound dimers. For simplicity, we adopt a single-mode approach (SMA) (Calsamiglia *et al.*, 2001; Vardi *et al.*, 2001) here to understand some main features of this nonlinear dynamical system, which is valid especially for small atomic kinetic energy or a magnetic pulse of short duration. In fact, the recent experiment

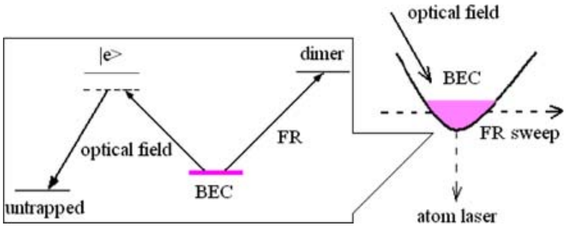

Fig. 1. (Color online) Schematic diagram of a Raman atom laser coupler by using a Feshbach-resonance-tuned atomic BEC. A new feature here is to consider the dimer creation in the trap triggered by a Feshbach sweep. In practice, to realize a FR-tuned source BEC and the FR-induced atom-dimer conversion in a single trapped BEC, a technique of gradient magnetic field can be used to realize different atomic scattering lengths (Xiong *et al.*, 2005).

of Winkler *et al.* on coherent dimer generation can be well described by SMA (Mackie *et al.*, 2000; Vanhaecke *et al.*, 2002; Winkler *et al.*, 2005; Wynar *et al.*, 2000). Here we focus on the couplings of different components and neglect the atomic collisions since their strength can be tuned by the FR method and our main purpose is to see the impact of atom-dimer coupling on the atomic output. The boson annihilation operators for the three-state atoms and the formed dimers are denoted by $\hat{b}_1$, $\hat{e}$, $\hat{b}_2$ and $\hat{g}$, respectively. Defining the optical/magnetic coupling strengths as G, κ and γ, the effective Hamiltonian in a rotating frame is then ($\hbar = 1$)

$$\mathcal{H}_{\text{int}} = -\delta\hat{e}^\dagger\hat{e} + G'(\hat{e}^\dagger\hat{b}_1 + \hat{b}_1^\dagger\hat{e}) + \kappa(\hat{b}_2^\dagger\hat{e} + \hat{e}^\dagger\hat{b}_2) + \gamma(\hat{g}^\dagger\hat{b}_1\hat{b}_1 + \hat{b}_1^\dagger\hat{b}_1^\dagger\hat{g}), \quad (1)$$

where δ is the atomic intermediate detuning and the coupling strengths are taken as real numbers. We neglect the kinetic-energy terms in the Hamiltonian and do not consider the the regime of long-distance atomic propagation. We note that, for this regime, by using the MF nonlinear Schrödinger equation, some effects were predicted by Zhang *et al.* for a travelling hybrid beam with large attractive atom-dimer interactions, such as the nonlocal behaviors of the matter waves (Zhang *et al.*, 2003b). Obviously there exists a conserved quantity for this dynamical system: $\sum\hat{b}_i^\dagger\hat{b}_i + \hat{e}^\dagger\hat{e} + 2\hat{g}^\dagger\hat{g} \equiv N_0$ ($i = 1, 2$), where N_0 is the total atom number for a condensate of all atoms or twice the total molecule numbers.

The Heisenberg equation of motion of the excited atoms, by assuming $|\delta|$ as the largest evolution parameters (Mackie *et al.*, 2000; Wynar *et al.*, 2000; Vanhaecke *et al.*, 2002; Winkler *et al.*, 2005) of the system or $\dot{\hat{e}}/\delta = 0$, leads to: $\hat{e} \approx (G'\hat{b}_1 + \kappa\hat{b}_2)/\delta$. Substituting this into Eq. (1) can adiabatically eliminate the excited atomic state, which yields an effective three-mode Hamiltonian in the form of Eq. (1) but only with one linear atomic coupling term ($G = \kappa G'/\delta$) : $G(\hat{b}_2^\dagger\hat{b}_1 + \hat{b}_1^\dagger\hat{b}_2)$. In this way, our model also can include a linear quasi-bond-bond coupling by applying another laser pulse, which again has the form of Eq. (1) but instead with an atom-stable-dimer coupling (Calsamiglia *et al.*, 2001; Mackie *et al.*, 2000; Vanhaecke *et al.*, 2002; Vardi *et al.*, 2001; Winkler *et al.*, 2005; Wynar *et al.*, 2000). Note that our Hamiltonian is a generalization of the well-known coherent two-color PA Hamiltonian (Mackie *et al.*, 2000; Vanhaecke *et al.*, 2002; Winkler *et al.*, 2005; Wynar *et al.*, 2000) under the adiabatic approximation. In addition, our nonlinear Hamiltonian can be further linearized as a perturbed three-level optics (TLO) model (Bergmann *et al.*, 1998; Eckert *et al.*, 2004; Greentree *et al.*, 2004; Haine and Hope, 2005; Poulsen and Molmer, 2001; Search, 2001) by using a rough mean-field (MF) approximation $(\hat{A} - \langle\hat{A}\rangle)(\hat{B} - \langle\hat{B}\rangle) \approx 0$, and its exact solutions can be achieved easily (not studied here).

The quantum dynamics and statistics of this nonlinear three-state system can be analytically studied under the short-time evolution limit. For this, we firstly write the Heisenberg equations of motion for the trapped and untrapped atoms, and the dimer modes as

$$\dot{\hat{b}}_1 = iG\hat{b}_2 + 2i\gamma\hat{b}_1^\dagger\hat{g}, \quad \dot{\hat{b}}_2 = iG\hat{b}_1, \quad \dot{\hat{g}} = i\gamma\hat{b}^2, \tag{2}$$

respectively. This is valid since the loss of atoms from a condensed state occurs in impressively short time scales (up to two hundreds of μs) (Anglin and Ketterle, 2002; Dalfovo *et al.*, 1999; Meystre, 2001). Thereby we now can readily derive the following solutions in second order of evolution time:

$$\begin{aligned}
\hat{b}_1(t) &= \hat{b}_1 + itG\hat{b}_2 + 2it\gamma\hat{b}_1^\dagger\hat{g} - \frac{1}{2}t^2G^2\hat{b}_1 + t^2G\gamma\hat{b}_2^\dagger\hat{g} \\
&\quad - t^2\gamma^2\hat{b}_1^\dagger\hat{b}_1^2 + 2t^2\gamma^2\hat{b}_1\hat{g}^\dagger\hat{g}, \\
\hat{b}_2(t) &= \hat{b}_2 + itG\hat{b}_1 - \frac{1}{2}t^2G^2\hat{b}_2 - t^2G\gamma\hat{b}_1^\dagger\hat{g}, \\
\hat{g}(t) &= \hat{g} + it\gamma\hat{b}_1^2 - t^2G\gamma\hat{b}_2\hat{b}_1 - t^2\gamma^2(2\hat{b}_1^\dagger\hat{b}_1 + 1)\hat{g}.
\end{aligned} \tag{3}$$

From this one can easily verify that $(i = 1, 2) : \sum \hat{b}_i^\dagger(t)\hat{b}_i(t) + 2\hat{g}^\dagger(t)\hat{g}(t) \equiv N_0$, as it should be. Note that the present scheme is different from our recent work on quantum superchemistry process in a pulsed atom laser coupler (Jing and Cheng, 2006). For the later, we considered a narrow optical PA sheet in a levitation trap instead of a source BEC trap and, by applying the fully quantum method of positive-P representation in quantum optics (Walls and Milburn, 1994), we predicted an optimal (stable) condition to separate the collected hybrid atom-molecule sample in the levitation-trap scheme first presented by Herbig *et al.* (2003; Donley *et al.*, 2002; Inouye *et al.*, 1998; Roberts *et al.*, 2001).

Now, with the solutions obtained above, we can readily study the quantum statistics of the coupled atom-dimer condensates. We will see that in the short-time limits, unlike the usual case of FR-induced dimer creation in a trapped

BEC, the formed dimers can exhibit an interesting squeezing-free property even in presence of the nonlinear atom-dimer coupling, which however can be greatly altered by considering the role of quantum statistics of the initial trapped atoms. As a convenient starting point, we consider an initial coherent system with a factorized structure as: $|in\rangle = |\alpha\rangle_1|0\rangle_2|0\rangle_g$ where $|\alpha\rangle$ is a Glauber coherent state, $\hat{b}_1|\alpha\rangle = |\alpha|\underline{e}^{i\varphi}$. The quadrature squeezing is determined by Walls and Milburn (1994)

$$\mathcal{S}_i = \frac{\langle(\Delta\hat{\mathcal{G}}_i)^2\rangle - \frac{1}{2}|\langle[\hat{\mathcal{G}}_1, \hat{\mathcal{G}}_2]\rangle}{\frac{1}{2}|\langle[\hat{\mathcal{G}}_1, \hat{\mathcal{G}}_2]\rangle|}, \quad i = 1, 2 \tag{4}$$

with $\hat{\mathcal{G}}_1 = \frac{1}{2}(\hat{O} + \hat{O}^\dagger)$ and $\hat{\mathcal{G}}_2 = \frac{1}{2i}(\hat{O} - \hat{O}^\dagger)$, by which we obtain the following simple result

$$\begin{pmatrix} S_{1b}(t) \\ S_{2b}(t) \end{pmatrix} = 3|\alpha|^2\lambda_1^2 t^2 \begin{pmatrix} \sin^2\varphi \\ \cos^2\varphi \end{pmatrix} > 0, \tag{5}$$

for the untrapped atoms. Similar result is obtained for the formed dimers but with a factor $|\alpha|^4\lambda_2^2 t^2$ and the replacement $\varphi \to 2\varphi$. This means that there is still no squeezing for these two species even beyond the Bogoliubov undepleted approximation. For the remained trapped atoms, it is reasonable to see that the dynamical quadrature squeezing usually can exist except for the case of $|\cos\varphi| = |\sin\varphi|$.

The quantum statistics of the initial trapped BEC also can play an important role in this system. Let us consider an initial squeezed BEC prepared by, e.g., the method of Orzel *et al.* (2001): $|\alpha\rangle_s = \hat{S}(\xi)|\alpha\rangle$, where $\hat{S}(\xi) = \exp[\xi(\hat{c}^\dagger)^2 - \xi^*\hat{c}^2]$, with $\xi = \frac{r}{2}e^{-i\phi_s}$, is a squeezed operator (Walls and Milburn, 1994). Then it is straightforward to reach a result for the ratio of average occupations of untrapped atoms and the formed dimers, i.e., $\langle N_g(t)\rangle_s = \eta(1 + 3\sinh^2 r)\langle N_b(t)\rangle_s$, here $\eta \equiv (\gamma/G)^2$. This indicates that the relative coupling-strength η and the squeezed strength r are two important parameters. For $\eta = 1$, we have $\bar{N}_g > \bar{N}_b$, more dimers are formed than the untrapped atoms, which can be even enhanced by a larger initial squeezing. And it is interesting that, the degree of BEC squeezing may be evaluated by measuring the relative occupations of the two species. Secondly, from the Eq. (4), we obtain the squeezed coefficients for the untrapped atoms as follows:

$$\mathcal{S}_{1b,2b}^{\xi_c}(t) = 2G^2t^2 \sinh^2 r(\sinh r \pm \cosh r \cos\phi_s), \tag{6}$$

and

$$\begin{pmatrix} \mathcal{S}_{1g}^{\xi_c}(\tau) \\ \mathcal{S}_{2g}^{\xi_c}(\tau) \end{pmatrix} = \gamma^2 t^2 \sinh^2 r \left[11(\sinh^2 r + 1) \begin{pmatrix} \cos^2 \phi_s \\ \sin^2 \phi_s \end{pmatrix} - 4 \right], \tag{7}$$

for the formed dimers. These results indicate an interesting ϕ_s-dependent squeezed effect: (i) for $\phi_s = 2n\pi (n = 0, 1, 2, \ldots)$, we have $\mathcal{S}_{1b} = G^2t^2(e^{2r} - 1) > 0$, $\mathcal{S}_{2b} = G^2t^2(e^{-2r} - 1) < 0$, which means that the quadrature component $\mathcal{S}_{2b}$ is squeezed; (ii) for $\phi_s = (2n + 1)\pi$, we have $\mathcal{S}_{1b} < 0, \mathcal{S}_{2b} > 0$, the quantum squeezing now transfers to $\mathcal{S}_{1b}$ component; (Note that, for the formed dimers, the squeezed effect will remain in the component $\mathcal{S}_{2g}$.) (iii) for $\phi_1 = (n + 1/2)\pi$, there is still squeezing for the dimers but never for the atoms; (iv) at last, for $\phi_1 = (n + 1/4)\pi$, no squeezing happens for the dimers (but for $r < \ln(1 + \sqrt{2}) \approx 0.88$, there is still atomic squeezing). In addition, we can easily verify that the Mandel's Q parameters (Walls and Milburn, 1994) for the two species are: $Q^s_{b,g}(\tau) \equiv \langle \Delta \hat{N}_g^2(\tau)\rangle_s / \langle \hat{N}_g(\tau)\rangle_s - 1 < 0$, which means that the two (atom-dimer) species both exhibit the non-classical sub-Poisson distributions. The mutual coherence of the two species can also be studied by evaluating the familiar second-order cross-correlation function (Walls and Milburn, 1994): $g^2_{bg}(t) < 1$ (anti-correlated states), hence the correlations between the two species can be dynamically established in this system. These rich quantum behaviors are quite different from the simple case with an initial coherent atomic BEC as describe above.

Finally we propose that, by applying a quantized optical photo-association (PA) field instead of a magnetic Feshbach sweep, a novel quantum transfer process and hence an atom-dimer entanglement may be expected in the present system. In fact, if we denote two quantized input lights as $\hat{a}_{1,2}$ for the atom-atom and atom-dimer couplings, respectively, under the large condensed atoms assumption: $\hat{b}_1, \hat{b}_1^\dagger \to \sqrt{N_0}$ the total Hamiltonian can then be linearized as

$$\mathcal{H}_4 = \mathcal{H}_0 + \lambda_1[\hat{b}_2^\dagger \hat{a}_1 + \hat{a}^\dagger \hat{b}_2] + \lambda_2[\hat{g}^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{g}],$$

where $\lambda_1 = G\sqrt{N_0}$, $\lambda_2 = \gamma\sqrt{N_0}$ and $\mathcal{H}_0$ denotes the free-part of the total Hamiltonian. Obviously, this simple linear form of Hamiltonian determines a factorized structure of wave function, especially a perfect quantum conversion process: $\hat{a}_1 \to \hat{b}$, $\hat{a}_2 \to \hat{g}$ (Bergmann *et al.*, 1998; Duan *et al.*, 2000; Eckert *et al.*, 2004; Greentree *et al.*, 2004; Haine and Hope, 2005; Poulsen and Molmer, 2001; Pu and Meystre, 2000; Search, 2001) according to which the quantum two-species entanglement can be realized by applying two entangled input lights. As a concrete example, we consider an optical two-mode squeezed state as: $|\zeta\rangle = \exp(\zeta \hat{a}_1^\dagger \hat{a}_1^\dagger - \zeta^* \hat{a}_1 \hat{a}_2)|0\rangle$, $\zeta = \kappa e^{-i\theta_s}$, we can easily show that, for $\lambda_1 = \lambda_2$, we have $g^{(2)}_{bg}(t) = 2 + \sinh^{-2}\lambda_1 > 1$ (correlated state) and, for the two atom-dimer species: $[g^{(2)}_{bg}(t)]^2 > g^{(2)}_b(t) g^{(2)}_g(t)$, i.e., there is a violation of the Cauchy-Schwarz inequality (CSI) which, according to Reid and Walls (1994), indicates a violation of Bell's inequality.

In conclusion, we have studied a Raman atom laser output coupler by using a FR-tuned atomic BEC with focus on the role of FR-sweep-induced dimer formation on the quantum dynamics of the system. This system can be adiabatically reduced into an effective three-state model for which the effects of quantum quadrature-squeezing and atom-dimer mutual coherence are observed. Also, as an interesting parallel of the scheme of Herbig *et al.* (2003; Donley *et al.*, 2002; Inouye *et al.*, 1998; Roberts *et al.*, 2001), this may shed a new light on another feasible method to separate the hybrid atom-dimer sample (Mackie *et al.*, 2000; Vanhaecke *et al.*, 2002; Winkler *et al.*, 2005; Wynar *et al.*, 2000) by utilizing a Raman-type output coupler. In practice, to realize the Raman atom laser output from a FR-tuned source BEC and the FR-induced atom-dimer conversion simultaneously, one could use a technique of gradient magnetic field with appropriate parameters which was recently proposed to generate a coexistence of different atomic scattering lengths in a single atomic condensate (Xiong *et al.*, 2005).

Of course, even more intriguing subjects can exist in this system. For example, the effects of particles collisions in propagating modes should be considered in further analysis. In fact, our earlier work on this problem within the context of an atom-laser out-coupling (Jing *et al.*, 2001) shows that the squeezed effects also can be predicted for the untrapped atoms due to their intrinsic collisions. For the quantized input lights including a quantized PA light (Jing and Zhan, 2006), the atomic depletions effect and the quantum noise terms should be studied by, e.g., the standard numerical method based on c-number stochastic equations in positive-P representation of quantum optics (Jing and Cheng, 2006; Walls and Milburn, 1994). Our investigations here provides the possibilities for these appealing researches.

ACKNOWLEDGMENTS

H. J. thanks Prof. Qi-run Dai for agreeing to visit his theory group in Xinyang where the idea and most parts of this work were created with his stimulating discussions and numerous helps. This research is financially supported by the NSFC, Wuhan Sunshine Plan and the CAS Liuxue Fund.

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